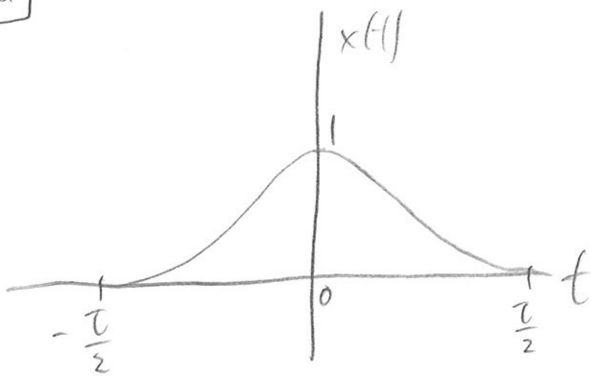
1.a



1.b

$$\cos^2\left(\pi\frac{t}{\tau}\right) = \frac{1}{2}\left(1+\cos\left(2\pi\frac{t}{\tau}\right)\right) = \frac{1}{2}\left(1+\frac{1}{2}\left(e^{+2\pi j\left(\frac{1}{\tau}\right)t} + e^{-2\pi j\left(\frac{1}{\tau}\right)t}\right)\right)$$

$$\Pi(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}(f)$$

$$\Pi\left(\frac{t}{\tau}\right) \xrightarrow{\mathcal{F}} \tau\,\mathrm{sinc}(\tau f)$$

$$\Pi\left(\frac{t}{\tau}\right)e^{2\pi j\left(\frac{1}{\tau}\right)t} \xrightarrow{\mathcal{F}} \tau\,\mathrm{sinc}\left(\tau\left(f-\frac{1}{\tau}\right)\right)$$

$$X(f) = \frac{1}{2}\left(\tau\,\mathrm{sinc}(\tau f) + \frac{\tau}{2}\mathrm{sinc}(\tau f-1) + \frac{\tau}{2}\mathrm{sinc}(\tau f+1)\right)$$

we could stop here, but if we continue to simplify, we can gain some insight into X(f).

$$\sin(\pi(\tau f \pm 1)) = \sin(\pi f\tau \pm \pi) = -\sin(\tau f)$$

$$X(f) = \frac{\tau}{2\pi}\sin(\pi\tau f)\left(\frac{1}{\tau f} - \frac{1/2}{\tau f-1} - \frac{1/2}{\tau f+1}\right)$$

$$= \frac{\tau}{2\pi}\sin(\pi\tau f)\left(\frac{1}{\tau f} - \frac{1}{2}\left(\frac{(\tau f+1)+(\tau f-1)}{(\tau f-1)(\tau f+1)}\right)\right)$$

$$= \frac{\tau}{2\pi}\sin(\pi\tau f)\left(\frac{1}{\tau f} - \frac{1}{2}\frac{2\tau f}{(\tau f)^2-1}\right) = \frac{\tau}{2\pi}\sin(\pi f\tau)\left(\frac{1}{\tau f} + \frac{\tau f}{1-(\tau f)^2}\right)$$

$$= \frac{\tau}{2\pi}\sin(\pi\tau f)\left(\frac{(1-(\tau f)^2)+(\tau f)^2}{(\tau f)(1-(\tau f)^2)}\right) = \frac{\tau}{2}\frac{\sin(\pi\tau f)}{\pi\tau f}\left(\frac{1}{1-(\tau f)^2}\right)$$

$$= \boxed{\frac{\tau}{2}\mathrm{sinc}(\tau f)\frac{1}{1-(\tau f)^2}}$$

clearly, this is just a filtered rectangle pulse. The filter transfer function is $\frac{1}{1-(\tau f)^2}$.

1.c

$$\int_{-\infty}^{\infty} X(f)\,df = x(0) = \boxed{1}$$

2.a

$2\cos^2(\pi t) = 1 + \cos(2\pi t)$, which has period 1

$j\sin(2\pi t)$ also has period 1, so $x(t)$ has period $\boxed{T_0 = 1}$

2.b

$$x(t) = 1 + \cos(2\pi t) + j\sin(2\pi t)$$
$$= 1 + e^{2\pi j t} = \sum_{k=-\infty}^{\infty} X[k]\, e^{2\pi j k t}$$
$$= 1\underbrace{e^{2\pi j 0 t}}_{=1} + 1\cdot e^{2\pi j t} = \cdots + X[-1]\,e^{2\pi j(-1)t} + X[0]\underbrace{e^{2\pi j(0)t}}_{=1} + X[1]\,e^{2\pi j(1)t} + \cdots$$

these must be equal, so
$$\boxed{X[k] = \begin{cases} 1 & k = 0,1 \\ 0 & \text{else} \end{cases} = \delta[k] + \delta[k-1]}$$

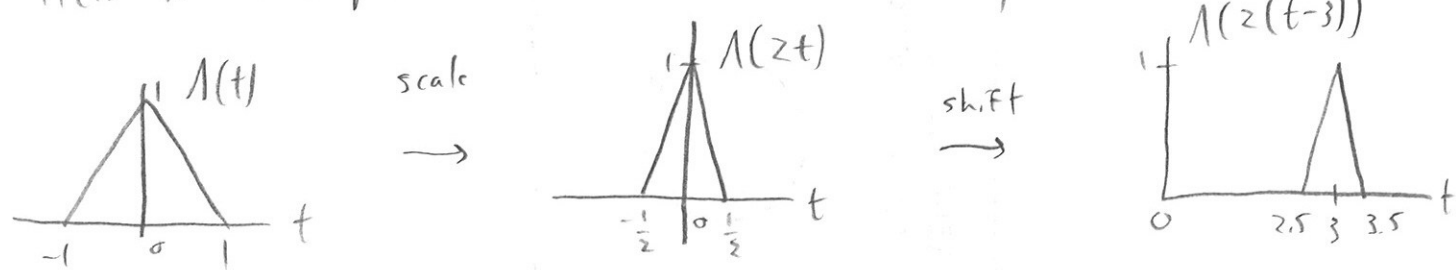
2.c

$$P\{x(t)\} = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |X[k]|^2 = 1^2 + 1^2 = \boxed{2}$$

this one looks a bit easier

3.a.

$H(f)$ is a couple of shifted and scaled triangles.



$$H(f) = j\Lambda(2(t-3)) - j\Lambda(2(t+3))$$

The impulse response is the inverse fourier transform of $H(f)$.

$$\Lambda(f) \xrightarrow{\mathcal{F}^{-1}} \text{sinc}^2(t)$$

$$\Lambda(2f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2}\text{sinc}^2\left(\frac{t}{2}\right)$$

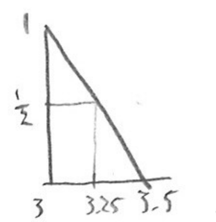
$$\Lambda(2(f\pm 3)) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2}\text{sinc}^2\left(\frac{t}{2}\right)e^{\pm 2\pi j 3t}$$

$$H(f) \xrightarrow{\mathcal{F}^{-1}} h(t) = j\frac{1}{2}\text{sinc}^2\left(\frac{t}{2}\right)e^{2\pi j 3t} - j\frac{1}{2}\text{sinc}^2\left(\frac{t}{2}\right)e^{-2\pi j 3t}$$

$$= -\frac{1}{2j}\left(e^{6\pi j t} - e^{-6\pi j t}\right)\text{sinc}^2\left(\frac{t}{2}\right)$$

$$= \boxed{-\sin(6\pi t)\,\text{sinc}^2\left(\frac{t}{2}\right)}$$

3.b



$$\cos\left(2\pi\frac{13}{4}t\right) = \frac{1}{2}\left(e^{+2\pi j f_0 t} + e^{-2\pi j f_0 t}\right), \quad f_0 = \frac{13}{4} = 3.25$$

These are eigenfunctions, with eigenvalues of $H(\pm f_0)$. The system is linear so we can find the individual outputs and sum them together.

$$H(f_0) = \frac{j}{2}, \quad H(-f_0) = -H(-f_0) = \frac{-j}{2}$$

$$y(t) = H(f_0)\frac{1}{2}e^{2\pi j f_0 t} + \frac{1}{2}H(-f_0)e^{-2\pi j f_0 t}$$

$$= \frac{j}{4}\left(e^{2\pi j f_0 t} - e^{-2\pi j f_0 t}\right) = -\frac{1}{2}\frac{1}{2j}\left(e^{2\pi j f_0 t} - e^{-2\pi j f_0 t}\right) = \boxed{-\frac{1}{2}\sin(2\pi f_0 t)}$$

4. This is a convolution integral: $\int_{-\infty}^{\infty} \text{sinc}^2(t-\lambda)\,\text{sinc}(\lambda)\,d\lambda = \text{sinc}^2(t) * \text{sinc}(t)$

$$\text{sinc}^2(t) * \text{sinc}(t) \xrightarrow{\mathcal{F}} \Lambda(f)\cdot\Pi(f) = \tfrac{1}{2}\Lambda(2f) + \tfrac{1}{2}\Pi(f)$$

$$\tfrac{1}{2}\Lambda(2f) + \tfrac{1}{2}\Pi(f) \xrightarrow{\mathcal{F}^{-1}} \tfrac{1}{2}\cdot\tfrac{1}{2}\,\text{sinc}^2\!\left(\tfrac{t}{2}\right) + \tfrac{1}{2}\,\text{sinc}(f)$$

$$= \boxed{\tfrac{1}{4}\,\text{sinc}^2\!\left(\tfrac{t}{2}\right) + \tfrac{1}{2}\,\text{sinc}(t)}$$